GO Math Unit 4
Ch. 9-10
Book Pages

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Are You Ready?

Whole Number Operations

EXAMPLE 270×83

$$\begin{array}{rl} 270 & \\ \times\ 83 & \\ \hline 810 & \leftarrow 3 \times 270 \\ +21{,}600 & \leftarrow 80 \times 270 \\ \hline 22{,}410 & \leftarrow (3 \times 270) + (80 \times 270) \end{array}$$

Find the product.

1. 992×16 **2.** 578×27 **3.** 839×65 **4.** 367×23

Use Repeated Multiplication

EXAMPLE

$5 \times 5 \times 5 \times 5$ — Multiply the first two factors.

25×5 — Multiply the result by the next factor.

125×5 — Multiply that result by the next factor.

625 — Continue until there are no more factors to multiply.

Find the product.

5. $7 \times 7 \times 7$ **6.** $3 \times 3 \times 3 \times 3$ **7.** $6 \times 6 \times 6 \times 6 \times 6$ **8.** $2 \times 2 \times 2 \times 2 \times 2 \times 2$

Division Facts

EXAMPLE $54 \div 9 = \blacksquare$

Think: 9 times what number equals 54?
$9 \times 6 = 54$

$54 \div 9 = 6$

So, $54 \div 9 = 6$.

Divide.

9. $20 \div 4$ **10.** $21 \div 7$ **11.** $42 \div 7$ **12.** $56 \div 8$

LESSON
9.1 Exponents

6.EE.1
Write and evaluate ... expressions involving whole-number exponents.

ESSENTIAL QUESTION

How do you use exponents to represent numbers?

EXPLORE ACTIVITY Real World 6.EE.1

Identifying Repeated Multiplication

A real-world problem may involve repeatedly multiplying a factor by itself.

A scientist observed the hourly growth of bacteria and recorded his observations in a table.

Time (h)	Total bacteria
0	1
1	2
2	$2 \times 2 =$ ☐
3	$2 \times 2 \times 2 =$ ☐
4	$2 \times 2 \times 2 \times 2 =$ ☐

After 2 hours, there are $2 \cdot 2 = ?$ bacteria.

A Complete the table. What pattern(s) do you see in the Total bacteria column?

B Complete each statement.

At 2 hours, the total is equal to the product of two 2s.

At 3 hours, the total is equal to the product of _____ 2s.

At 4 hours, the total is equal to the product of _____ 2s.

Reflect

1. **Communicate Mathematical Ideas** How is the time, in hours, related to the number of times 2 is used as a factor?

Using Exponents

A number that is formed by repeated multiplication of the same factor is called a **power**. You can use an *exponent* and a *base* to write a power. For example, 7^3 means the product of three 7s:

$$7^3 = 7 \times 7 \times 7$$

The **base** is the number that is multiplied.

The **exponent** tells how many times the base appears in the expression.

Power	How to read the power
6^2	6 squared, 6 to the power of 2, 6 raised to the 2nd power
7^3	7 cubed, 7 to the power of 3, 7 raised to the 3rd power
9^4	9 to the power of 4, 9 raised to 4th power

EXAMPLE 1

COMMON CORE 6.EE.1

Use an exponent to write each expression.

A $3 \times 3 \times 3 \times 3 \times 3$

Find the base, or the number being multiplied. The base is 3.

Find the exponent by counting the number of 3s being multiplied. The exponent is 5.

$\underbrace{3 \times 3 \times 3 \times 3 \times 3}_{\text{5 factors of 3}} = 3^5$

B $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

Find the base, or the number being multiplied. The base is $\frac{4}{5}$.

Find the exponent by counting the number of times $\frac{4}{5}$ appears in the expression. The exponent is 4.

$\underbrace{\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}}_{\text{4 factors of } \frac{4}{5}} = \left(\frac{4}{5}\right)^4$

Math Talk
Mathematical Practices
What is the value of a number raised to the power of 1?

YOUR TURN

Use exponents to write each expression.

2. $4 \times 4 \times 4$ _____
3. 6 _____
4. $\frac{1}{8} \times \frac{1}{8}$ _____
5. $5 \times 5 \times 5 \times 5 \times 5 \times 5$ _____

Finding the Value of a Power

To find the value of a power, remember that the exponent indicates how many times to use the base as a factor.

Property of Zero as an Exponent

The value of any nonzero number raised to the power of 0 is 1.

Example: $5^0 = 1$

EXAMPLE 2

COMMON CORE 6.EE.1

Find the value of each power.

A 10^4

Identify the base and the exponent.
The base is 10, and the exponent is 4.

Evaluate: $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

B 0.4^3

Identify the base and the exponent.
The base is 0.4, and the exponent is 3.

Evaluate: $0.4^3 = 0.4 \times 0.4 \times 0.4 = 0.064$

C $\left(\frac{3}{5}\right)^0$

Identify the base and the exponent.
The base is $\frac{3}{5}$, and the exponent is 0.

Evaluate.

$\left(\frac{3}{5}\right)^0 = 1$ Any number raised to the power of 0 is 1.

D $\left(\frac{2}{3}\right)^2$

Identify the base and the exponent.
The base is $\frac{2}{3}$, and the exponent is 2.

Evaluate.

$\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) = \frac{4}{9}$

Math Talk
Mathematical Practices
Is the value of 2^3 the same as the value of 3^2? Explain.

YOUR TURN

Find the value of each power.

6. 3^4 _____
7. $(1)^9$ _____
8. $\left(\frac{2}{5}\right)^3$ _____
9. 12^2 _____

Guided Practice

1. Complete the table. (Explore Activity 1)

Exponential form	Product	Simplified product
5^1	5	5
5^2	5 × 5	
5^3		125
	5 × 5 × 5 × 5	
5^5		

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Selected Answers — See all the selected answers.

Use an exponent to write each expression. (Example 1)

2. 6 × 6 × 6 _____ **3.** 10 × 10 × 10 × 10 × 10 × 10 × 10 _____

__ factors of 6

4. $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ _____ **5.** $\frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9}$ _____

Find the value of each power. (Example 2)

6. 8^3 _____ **7.** 7^4 _____ **8.** 10^3 _____

9. $\left(\frac{1}{4}\right)^2$ _____ **10.** $\left(\frac{1}{3}\right)^3$ _____ **11.** $\left(\frac{6}{7}\right)^2$ _____

12. 0.8^2 _____ **13.** 0.5^3 _____ **14.** 1.1^2 _____

15. 8^0 _____ **16.** 12^1 _____ **17.** $\left(\frac{1}{2}\right)^0$ _____

18. $(13)^2$ _____ **19.** $\left(\frac{2}{5}\right)^2$ _____ **20.** 0.9^2 _____

ESSENTIAL QUESTION CHECK-IN

21. How do you use an exponent to represent a number such as 16?

_____

_____

_____

_____

_____

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9.1 Independent Practice

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Selected Answers — See all the selected answers.

COMMON CORE 6.EE.1

Write the missing exponent.

22. $100 = 10^{\square}$ **23.** $8 = 2^{\square}$ **24.** $25 = 5^{\square}$ **25.** $27 = 3^{\square}$

26. $\frac{1}{169} = \left(\frac{1}{13}\right)^{\square}$ **27.** $14 = 14^{\square}$ **28.** $32 = 2^{\square}$ **29.** $\frac{64}{81} = \left(\frac{8}{9}\right)^{\square}$

Write the missing base.

30. $1{,}000 = \square^3$ **31.** $256 = \square^4$ **32.** $16 = \square^4$ **33.** $9 = \square^2$

34. $\frac{1}{9} = \left(\square\right)^2$ **35.** $64 = \square^2$ **36.** $\frac{9}{16} = \left(\square\right)^2$ **37.** $729 = \square^3$

For Exercises 38–42, write the answer with and without using an exponent.

38. Hadley's softball team has a phone tree in case a game is canceled. The coach calls 3 players. Then each of those players calls 3 players, and so on. How many players will be notified during the third round of calls?

_____

39. Tim is reading a book. On Monday he reads 3 pages. On each day after that, he reads 3 times the number of pages that he read on the previous day. How many pages does he read on Thursday?

_____

40. The square tile shown has a side length of 10.5 inches. What power can you write to represent the area of the tile? Write the power as an expression with a base and an exponent, and then find the area of the square.

_____

41. Antonia is saving for a video game. On the first day, she saves two dollars in her piggy bank. Each day after that, she doubles the number of dollars she saved on the previous day. How many dollars does she save on the sixth day?

_____

42. A certain colony of bacteria triples in length every 10 minutes. Its length is now 1 millimeter. How long will it be in 40 minutes?

_____

LESSON
9.2 Prime Factorization

6.EE.1
Write and evaluate numerical expressions involving whole-number exponents.

ESSENTIAL QUESTION

How do you write the prime factorization of a number?

EXPLORE ACTIVITY Real World

Prep for 6.EE.1

Finding Factors of a Number

Whole numbers that are multiplied to find a product are called factors of that product. A number is divisible by its factors. 8 has factors 1, 2, 4, and 8 because $4 \cdot 2 = 8$, and 8 is divisible by 1, 2, 4, and 8.

ADD A NOTE

Math On the Spot

EXAMPLE 1 **Ana wants to build a rectangular garden with an area of 24 square feet. What are the possible whole number lengths and widths of the garden?**

STEP 1 Recall that area = length · width. For Ana's garden, _____ ft^2 = length · width.

STEP 2 List the factors of 24 in pairs. List each pair only once.

$24 = 1 \cdot$ _____ $24 = 3 \cdot$ _____

$24 = 2 \cdot$ _____ $24 = 4 \cdot$ _____

$4 \cdot 6 = 6 \cdot 4$, so you only list $4 \cdot 6$.

You can also use a diagram to show the factor pairs.

1 2 3 4 ☐ ☐ ☐ ☐

The factors of 24 are: 1, 2, 3, _____.

STEP 3 The possible lengths and widths are:

Length (ft)	24	12	8	6
Width (ft)				

YOUR TURN

List all the factors of each number.

1. 21 _____ **2.** 37 _____

3. 42 _____ **4.** 30 _____

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Your Turn

EXPLORE ACTIVITY 2 6.EE.1

Finding the Prime Factorization of a Number

The prime factorization of a number is the number written as the product of its prime factors. For example, the prime factors of 12 are 3, 2, and 2.

The prime factorization of 12 is $2 \cdot 3 \cdot 2$ or $2^2 \cdot 3$.

Use exponents to show repeated factors.

Animated Math

Use a factor tree to find the prime factorization of 240.

A List the factor pairs of 240.

240
· 48

_____

_____

B Choose any factor pair to begin the tree. If a number in this pair is prime, circle it. If a number in the pair can be written as a product of two factors, draw additional branches and write the factors.

C Continue adding branches until the factors at the ends of the branches are prime numbers.

D Write the prime factorization of 240.

_____

Then write the prime factorization using exponents.

_____

240

Reflect

5. **What If?** What will the factor tree for 240 look like if you start the tree with a different factor pair? Check your prediction by creating another factor tree for 240 that starts with a different factor pair.

_____

_____

_____

_____

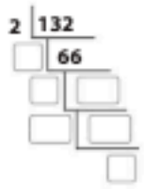
EXPLORE ACTIVITY 3 COMMON CORE 6.EE.1

Using a Ladder Diagram

A ladder diagram is another way to find the prime factorization of a number.

Use a ladder diagram to find the prime factorization of 132.

A Write 132 in the top "step" of the ladder. Choose a prime factor of 132 to write next to the step with 132. Choose 2. Divide 132 by 2 and write the quotient 66 in the next step of the ladder.



B Now choose a prime factor of 66. Write the prime factor next to the step with 66. Divide 66 by that prime factor and write the quotient in the next step of the ladder.

C Keep choosing prime factors, dividing, and adding to the ladder until you get a quotient of 1.

D What are the prime factors of 132? How can you tell from the ladder diagram?

_____

_____

E Write the prime factorization of 132 using exponents.

_____

_____

Reflect

6. Complete a factor tree and a ladder diagram to find the prime factorization of 54.

7. **Communicate Mathematical Ideas** If one person uses a ladder diagram and another uses a factor tree to write a prime factorization, will they get the same result? Explain.

_____

_____

Guided Practice

Use a diagram to list the factor pairs of each number. (Explore Activity Example 1)

1. 18

2. 52

_____ _____

3. Karl needs to build a stage that has an area of 72 square feet. The length of the stage should be longer than the width. What are the possible whole number measurements for the length and width of the stage? (Explore Activity Example 1)

Complete the table with possible measurements of the stage.

Length	72					
Width		2				

Use a factor tree to find the prime factorization of each number. (Explore Activity 2)

4. 402

5. 36

_____ _____

Use a ladder diagram to find the prime factorization of each number. (Explore Activity 3)

6. 64

7. 27

_____ _____

ESSENTIAL QUESTION CHECK-IN

8. Tell how you know when you have found the prime factorization of a number.

_____

_____

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9.2 Independent Practice

COMMON CORE 6.EE.1

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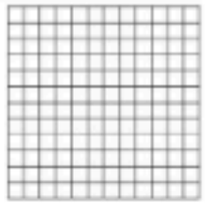
Selected Answers

9. **Multiple Representations** Use the grid to draw three different rectangles so that each has an area of 12 square units and they all have different widths. What are the dimensions of the rectangles?



_____

_____

10. Brandon has 32 stamps. He wants to display the stamps in rows, with the same number of stamps in each row. How many different ways can he display the stamps? Explain.

_____

_____

_____

11. **Communicate Mathematical Ideas** How is finding the factors of a number different from finding the prime factorization of a number?

_____

_____

_____

_____

Find the prime factorization of each number.

12. 891 _____ **13.** 504 _____

14. 23 _____ **15.** 230 _____

16. The number 2 is chosen to begin a ladder diagram to find the prime factorization of 66. What other numbers could have been used to start the ladder diagram for 66? How does starting with a different number change the diagram?

_____

_____

_____

17. **Critical Thinking** List five numbers that have 3, 5, and 7 as prime factors.

_____

LESSON

9.3 Order of Operations

COMMON CORE 6.EE.1

Write and evaluate ... expressions involving whole-number exponents.

ESSENTIAL QUESTION

How do you use the order of operations to simplify expressions with exponents?

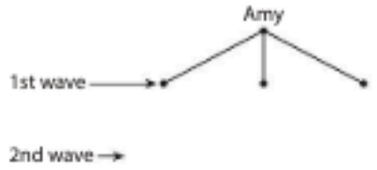
EXPLORE ACTIVITY Real World COMMON CORE 6.EE.1

Exploring the Order of Operations

Order of Operations

1. Perform operations in parentheses.
2. Find the value of numbers with exponents.
3. Multiply or divide from left to right.
4. Add or subtract from left to right.

Amy and three friends launch a new website. Each friend e-mails the web address to three new friends. These new friends forward the web address to three more friends. If no one receives the e-mail more than once, how many people will receive the web address in the second wave of e-mails?



A Use a diagram to model the situation for Amy. Each dot represents one e-mail. Complete the diagram to show the second wave.

B Complete the table to show how many e-mails are sent in each wave of Amy's diagram.

Wave	Number of e-mails	Power of 3
1^{st}		
2^{nd}		

C Amy is just one of four friends initiating the first wave of e-mails. Write an expression for the total number of e-mails sent in the 2nd wave.

number of people × number of e-mails in 2nd wave written as a power

□ × □

D Identify the computation that should be done first to simplify the expression in C. Then simplify the expression.

Multiply 4 and 3 / Find the value of 3^2

The value of the expression is 4 × _____ = _____.

EXPLORE ACTIVITY (cont'd)

Reflect

1. In B, why does it makes sense to write the numbers of e-mails as powers? What is the pattern for the number of e-mails in each wave for Amy?

_____

_____

_____

Simplifying Numerical Expressions

A numerical expression is an expression involving numbers and operations. You can use the order of operations to simplify numerical expressions.

EXAMPLE 1

6.EE.1

Simplify each expression.

A $5 + 18 \div 3^2$

$5 + 18 \div 3^2 = 5 + 18 \div 9$ Evaluate 3^2.

$= 5 + 2$ Divide.

$= 7$ Add.

B $21 + \frac{3^2}{3}$

$21 + \frac{3^2}{3} = 21 + \frac{9}{3}$ Evaluate 3^2.

$= 21 + 3$ Divide.

$= 24$ Add.

C $6 \times 2^3 \div 3 + 1$

$6 \times 2^3 \div 3 + 1 = 6 \times 8 \div 3 + 1$ Evaluate 2^3.

$= 48 \div 3 + 1$ Multiply.

$= 16 + 1$ Divide.

$= 17$ Add.

YOUR TURN

Simplify each expression using the order of operations.

2. $7 + 15 \times 9^2 =$ _____

3. $220 - 450 \div 3^2 =$ _____

Using Exponents with Grouping Symbols

Remember to perform operations inside parentheses first when you simplify expressions.

EXAMPLE 2

6.EE.1

Simplify each expression using the order of operations.

A $4 \times (9 \div 3)^2$

$4 \times (9 \div 3)^2 = 4 \times 3^2$ Perform operations inside parentheses.

$= 4 \times 9$ Evaluate 3^2.

$= 36$ Multiply.

B $5^3 + (12 - 2)^2$

$5^3 + (12 - 2)^2 = 5^3 + 10^2$ Perform operations inside parentheses.

$= 125 + 100$ Evaluate powers.

$= 225$ Add.

C $8 + \frac{(12 - 8)^2}{2}$

$8 + \frac{(12 - 8)^2}{2} = 8 + \frac{4^2}{2}$ Perform operations inside parentheses.

$= 8 + \frac{16}{2}$ Evaluate 4^2.

$= 8 + 8$ Divide.

$= 16$ Add.

Reflect

4. **Critique Reasoning** John wants to simplify the expression $(5 + 3)^2$. As a first step, he writes $5^2 + 3^2$. Will he get the correct value for the expression? If not, what should he do to simplify the expression?

_____

_____

_____

YOUR TURN

Simplify each expression using the order of operations.

5. $5 \times (20 \div 4)^2 =$ _____

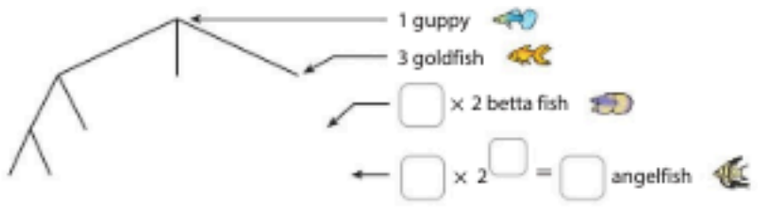
6. $8^2 - (5 + 2)^3 =$ _____

7. $7 - \frac{(63 \div 9)^2}{7} =$ _____

My Notes

Guided Practice

1. In a video game, a guppy that escapes a net turns into three goldfish. Each goldfish can turn into two betta fish. Each betta fish can turn into two angelfish. Complete the diagram and write the number of fish at each stage. Write and evaluate an expression for the number of angelfish that can be formed from one guppy. (Explore Activity)



_____

Complete to simplify each expression. (Examples 1 and 2)

2. $89 - 4^2 \times 4 + 12 = 89 - \_\_\_\_ \times 4 + 12$

$= 89 - \_\_\_\_ + 12$

$= \_\_\_\_ + 12$

$= \_\_\_\_$

3. $6 \times (36 \div 12)^2 + 8 = 6 \times (\_\_\_\_)^2 + 8$

$= 6 \times \_\_\_\_ + 8$

$= \_\_\_\_ + 8$

$= \_\_\_\_$

4. $12 \times \left(\frac{(4+2)^2}{4}\right) - 7 = 12 \times \left(\frac{\square^2}{4}\right) - 7$

$= 12 \times \left(\frac{\square}{4}\right) - 7$

$= 12 \times \_\_\_\_ - 7$

$= \_\_\_\_ - 7$

$= \_\_\_\_$

5. $320 \div \left(\frac{(11-9)^3}{2}\right) \times 8 = 320 \div \left(\frac{\square^3}{2}\right) \times 8$

$= 320 \div \left(\frac{\square}{2}\right) \times 8$

$= 320 \div \_\_\_\_ \times 8$

$= \_\_\_\_ \times 8$

$= \_\_\_\_$

ESSENTIAL QUESTION CHECK-IN

6. How do you use the order of operations to simplify expressions with exponents?

_____

_____

_____

Name _____ Class _____ Date _____

9.3 Independent Practice

COMMON CORE 6.EE.1

Simplify each expression using the order of operations.

7. $5 \times 2 + 3^2$ _____

8. $15 - 7 \times 2 + 2^3$ _____

9. $(11 - 8)^3 - 2 \times 6$ _____

10. $6 + 3(13 - 2) - 5^2$ _____

11. $12 + \frac{9^2}{3}$ _____

12. $\frac{8 + 6^2}{11} + 7 \times 2$ _____

13. **Explain the Error** Jay simplified the expression $3 \times (3 + 12 \div 3) - 4$. For his first step, he added $3 + 12$ to get 15. What was Jay's error? Find the correct answer.

_____

_____

14. **Multistep** A clothing store has the sign shown in the shop window. Pani sees the sign and wants to buy 3 shirts and 2 pairs of jeans. The cost of each shirt before the discount is \$12, and the cost of each pair of jeans is \$19 before the discount.

a. Write and simplify an expression to find the amount Pani pays if a \$3 discount is applied to her total.

_____

b. Pani says she should get a \$3 discount on the price of each shirt and a \$3 discount on the price of each pair of jeans. Write and simplify an expression to find the amount she would pay if this is true.

_____

c. **Analyze Relationships** Why are the amounts Pani pays in **a** and **b** different?

_____

_____

d. If you were the shop owner, how would you change the sign? Explain.

_____

_____

_____

_____

MODULE QUIZ

Ready to Go On?

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Ready to Go On?

9.1 Exponents

Find the value of each power.

1. 7^3 _____ **2.** 9^2 _____ **3.** $\left(\frac{7}{9}\right)^2$ _____ **4.** $\left(\frac{1}{2}\right)^6$ _____

5. $\left(\frac{2}{3}\right)^3$ _____ **6.** $\left(\frac{1}{3}\right)^4$ _____ **7.** 12^0 _____ **8.** 1.4^2 _____

9.2 Prime Factorization

Find the factors of each number.

9. 96 _____

10. 120 _____

Find the prime factorization of each number.

11. 58 _____ **12.** 212 _____

13. 2,800 _____ **14.** 900 _____

9.3 Order of Operations

Simplify each expression using the order of operations.

15. $(21 - 3) \div 3^2$ _____ **16.** $7^2 \times (6 \div 3)$ _____

17. $17 + 15 \div 3 - 2^4$ _____ **18.** $(8 + 56) \div 4 - 3^2$ _____

19. The nature park has a pride of 7 adult lions and 4 cubs. The adults eat 6 pounds of meat each day and the cubs eat 3 pounds. Simplify $7 \times 6 + 4 \times 3$ to find the amount of meat consumed each day by the lions. _____

ESSENTIAL QUESTION

20. How do you use numerical expressions to solve real-world problems?

_____

_____

_____

_____

COMMON CORE

MODULE 9 MIXED REVIEW

Assessment Readiness

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Selected Response

1. Which expression has a value that is less than the base of that expression?

Ⓐ 2^3
Ⓑ $\left(\frac{5}{6}\right)^7$
Ⓒ 3^2
Ⓓ 4^4

2. After the game the coach bought 9 chicken meals for $5 each and 15 burger meals for $6 each. What percent of the total amount the coach spent was used for the chicken meals?

Ⓐ $33\frac{1}{3}\%$
Ⓑ 45%
Ⓒ $66\frac{2}{3}\%$
Ⓓ 90%

3. Which operation should you perform first when you simplify $175 - (8 + 45 \div 3) \times 7$?

Ⓐ addition
Ⓑ division
Ⓒ multiplication
Ⓓ subtraction

4. For a game, three people are chosen in the first round. Each of those people chooses 3 people in the second round, and so on. How many people are chosen in the sixth round?

Ⓐ 18
Ⓑ 216
Ⓒ 243
Ⓓ 729

5. Which expression shows the prime factorization of 100?

Ⓐ $2^2 \times 5^2$ Ⓒ 10^{10}
Ⓑ 10×10 Ⓓ $2 \times 5 \times 10$

6. Which number has only two factors?

Ⓐ 21 Ⓒ 25
Ⓑ 23 Ⓓ 27

7. Which expression is equivalent to $3.6 \times 3.6 \times 3.6 \times 3.6$?

Ⓐ 3.6×4 Ⓒ $3^4 \times 6^4$
Ⓑ 36^3 Ⓓ 3.6^4

8. Which expression gives the prime factorization of 80?

Ⓐ $2^4 \times 10$
Ⓑ $2 \times 5 \times 8$
Ⓒ $2^3 \times 5$
Ⓓ $2^4 \times 5$

Mini-Task

9. George wants to put carpeting in a rectangular living room and a square bedroom. The length and width of the living room is 12 feet by 18 feet. One side of the square bedroom is 13 feet. It will cost $3.50 per square foot to carpet the rooms.

a. Write an expression that can be used to find the total amount George will pay for carpeting.

_____

b. Evaluate the expression. How much will George pay for the carpeting?

_____

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Use of Parentheses

EXAMPLE $(6 + 4) \times (3 + 8 + 1) = 10 \times 12$ — Do the operations inside parentheses first.

$= 120$ — Multiply.

Evaluate.

1. $11 + (20 - 13)$ _____

2. $(10 - 7) - (14 - 12)$ _____

3. $(4 + 17) - (16 - 9)$ _____

4. $(23 - 15) - (18 - 13)$ _____

5. $8 \times (4 + 5 + 7)$ _____

6. $(2 + 3) \times (11 - 5)$ _____

Words for Operations

EXAMPLE Write a numerical expression for the quotient of 20 and 5. — Think: *Quotient* means to divide.

$20 \div 5$ — Write 20 divided by 5.

Write a numerical expression for the word expression.

7. the difference between 42 and 19 _____

8. the product of 7 and 12 _____

9. 30 more than 20 _____

10. 100 decreased by 77 _____

Evaluate Expressions

EXAMPLE Evaluate $2(5) - 3^2$.

$2(5) - 3^2 = 2(5) - 9$ — Evaluate exponents.

$= 10 - 9$ — Multiply.

$= 1$ — Subtract.

Evaluate the expression.

11. $3(8) - 15$ _____

12. $4(12) + 11$ _____

13. $3(7) - 4(2)$ _____

14. $4(2 + 3) - 12$ _____

15. $9(14 - 5) - 42$ _____

16. $7(8) - 5(8)$ _____

LESSON 10.1 Modeling and Writing Expressions

COMMON CORE 6.EE.2a

Write expressions that record operations with numbers and with letters standing for numbers. *Also 6.EE.2b, 6.EE.4, 6.EE.6*

ESSENTIAL QUESTION

How can you model and write algebraic expressions?

EXPLORE ACTIVITY

COMMON CORE 6.EE.2a, 6.EE.2b

Writing Algebraic Expressions

An **algebraic expression** is an expression that contains one or more variables and may also contain operation symbols, such as + or −.

A **variable** is a letter or symbol used to represent an unknown or unspecified number. The value of a variable may change.

A **constant** is a specific number whose value does not change.

150 is a constant and y is a variable.

Algebraic Expressions	x	$w + n$	$150 + y$

In algebraic expressions, multiplication and division are usually written without the symbols × and ÷.

- Write $3 \times n$ as $3n$, $3 \cdot n$, or $n \cdot 3$.
- Write $3 \div n$ as $\frac{3}{n}$.

There are several different ways to describe expressions with words.

Operation	Addition	Subtraction	Multiplication	Division
Words	• added to • plus • sum • more than	• subtracted from • minus • difference • less than	• times • multiplied by • product • groups of	• divided by • divided into • quotient

EXAMPLE 1 **Complete each statement.**

A **Write each phrase as an algebraic expression.**

The sum of 7 and x

The operation is _____. The algebraic expression is _____.

The quotient of z and 3

The operation is _____. The algebraic expression is $\frac{\square}{\square}$.

EXPLORE ACTIVITY *(cont'd)*

B **Write a phrase for each expression.**

$11x$ The operation is _____.

Phrase: _____

$8 - y$ The operation is _____.

Phrase: _____

YOUR TURN

Write each phrase as an algebraic expression.

1. n times 7 _____ **2.** 4 minus y _____ **3.** 13 added to x _____

Write a phrase for each expression.

4. $\frac{x}{12}$ _____

5. $10y$ _____

6. $c + 3$ _____

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Your Turn

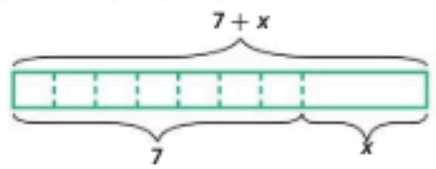
Modeling Algebraic Expressions

Algebraic expressions can also be represented with models.

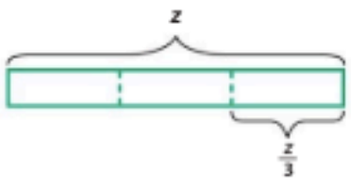
EXAMPLE 2 6.EE.2a

Use a bar model to represent each expression.

A $7 + x$ *Combine 7 and x.*



B $\frac{z}{3}$ *Divide z into 3 equal parts.*



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Interactive Example

YOUR TURN

Draw a bar model to represent each expression.

7. $t - 2$ **8.** $4y$

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Your Turn

Comparing Expressions Using Models

Algebraic expressions are *equivalent* if they are equal for all values of the variable. For example, $x + 2$ and $x + 1 + 1$ are equivalent.

EXAMPLE 3 Real World 6.EE.4

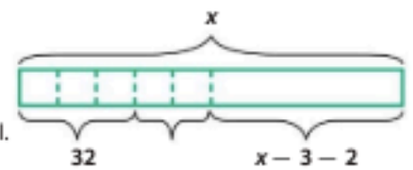
Katriana and Andrew started the day with the same amount of money. Katriana spent 5 dollars on lunch. Andrew spent 3 dollars on lunch and 2 dollars on a snack after school. Do Katriana and Andrew have the same amount of money left?

STEP 1 Write an algebraic expression to represent the money Katriana has left. Represent the expression with a model.

$x - 5$

The variable represents the amount of money both Katriana and Andrew have at the beginning of the day.

STEP 2 Write an algebraic expression to represent the money Andrew has left. Represent the expression with a model.



$x - 3 - 2$

STEP 3 Compare the models.

The models are equivalent, so the expressions are equivalent.

Andrew and Katriana have the same amount of money left.

Math On the Spot

My Notes

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Interactive Example

YOUR TURN

9. On a math quiz, Tina scored 3 points more than Julia. Juan scored 2 points more than Julia and earned 2 points in extra credit. Write an expression and draw a bar model to represent Tina's score and Juan's score. Did Tina and Juan make the same grade on the quiz? Explain.

Modeling Real-World Situations

You can use expressions to represent real-world situations.

EXAMPLE 4 Real World COMMON CORE 6.EE.6

A Tickets to the water park cost $53 per person. Write an expression to show the total cost of tickets for a group of people.

A group of is a clue to multiply. The ticket price of $53 is a constant. The number of people who need tickets is a variable.

Use x for the number of people.

The algebraic expression for the total cost of tickets is $53x$.

B Genise has some savings. After babysitting, she adds $75 to her savings. How much money has Genise saved?

Adds is a clue to use addition. The $75 Genise added to her savings is a constant. The amount of money Genise had saved before is unknown, so use a variable.

Use y for Genise's savings before she adds the babysitting money.

The algebraic expression for Genise's total savings is $y + 75$.

Math Talk
Mathematical Practices
How do you know what operation to use to find the amount each niece receives?

YOUR TURN

10. Helen divides up some money to give equally to her four nieces. If d represents the total amount, write an expression to represent how much money each niece receives. _____

Guided Practice

Write each phrase as an algebraic expression. (Explore Activity Example 1)

1. 3 less than y _____ **2.** The product of 2 and p _____

Write a phrase for each algebraic expression. (Explore Activity Example 1)

3. $y + 12$ _____ **4.** $\frac{p}{10}$ _____

5. Draw a bar model to represent the expression $m \div 4$. (Example 2)

At 6 p.m., the temperature in Phoenix, AZ, t, is the same as the temperature in Tucson, AZ. By 9 p.m., the temperature in Phoenix has dropped 2 degrees and in Tucson it has dropped 4 degrees. By 11 p.m., the temperature in Phoenix has dropped another 3 degrees. (Example 3)

6. Represent each city's temperature at 11 p.m. with an algebraic expression and a bar model.

t t

_____ _____

7. Are the expressions that represent the temperatures in the two cities equivalent? Justify your answer.

_____

_____

8. Noelle bought some boxes of water bottles for a picnic. Each box contained 24 bottles of water. If c is the number of boxes, write an expression to show how many bottles of water Noelle bought. (Example 4)

_____

ESSENTIAL QUESTION CHECK-IN

9. Give an example of a real-world situation that could be represented by an algebraic expression.

_____

_____

_____

Name _____ Class _____ Date _____

10.1 Independent Practice

6.EE.2a, 6.EE.2b, 6.EE.4, 6.EE.6

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Selected Answers — See all the selected answers.

10. Write an algebraic expression with the constant 7 and the variable *y*.

_____

Write each phrase as an algebraic expression.

11. *n* divided by 8 _____

12. *p* multiplied by 4 _____

13. *b* plus 14 _____

14. 90 times *x* _____

15. *a* take away 16 _____

16. *k* less than 24 _____

17. 3 groups of *w* _____

18. the sum of 1 and *q* _____

19. the quotient of 13 and *z* _____

20. *c* added to 45 _____

21. 8 less than *w* _____

Write a phrase in words for each algebraic expression.

22. $m + 83$ _____

23. $42s$ _____

24. $\frac{9}{d}$ _____

25. $t - 29$ _____

26. $2 + g$ _____

27. $11x$ _____

28. $\frac{h}{12}$ _____

29. $5 - k$ _____

Sarah and Noah work at Read On Bookstore and get paid the same hourly wage. The table shows their work schedule for last week.

Read On Bookstore Work Schedule (hours)			
	Monday	Tuesday	Wednesday
Sarah	5	3	
Noah			8

30. Write an expression that represents Sarah's total pay last week. Represent her hourly wage with *w*. _____

31. Write an expression that represents Noah's total pay last week. Represent his hourly wage with *w*. _____

32. Are the expressions equivalent? Did Sarah and Noah earn the same amount last week? Use models to justify your answer.

_____

33. Mia buys 3 gallons of gas that costs *d* dollars per gallon. Bob buys *g* gallons of gas that costs $3 per gallon.

a. Write an expression for the amount Mia pays for gas. _____

b. Write an expression for the amount Bob pays for gas. _____

c. What do the numeral and the variable represent in each expression?

_____

_____

_____

_____

LESSON 10.2

Evaluating Expressions

6.EE.2c

Evaluate expressions at specific values of their variables.

ESSENTIAL QUESTION

How can you use the order of operations to evaluate algebraic expressions?

EXPLORE ACTIVITY

6.EE.2c

Evaluating Expressions

Recall that an algebraic expression contains one or more variables. You can substitute a number for that variable and then find the value of the expression. This is called **evaluating** the expression.

Math on the Spot — Math On the Spot

EXAMPLE 1 **Evaluate each expression for the given value of the variable.**

A $x - 9; x = 15$

Substitute 15 for *x*. $\square - 9$

Subtract. $\square$

When $x =$ _____, $x - 9 =$ _____.

B $\frac{16}{n}; n = 8$

Substitute 8 for *n*. $\frac{16}{\square}$

Divide. $\square$

When $n =$ _____, $\frac{16}{n} =$ _____.

C $0.5y; y = 1.4$

Substitute 1.4 for *y*. $0.5(\square)$

Multiply. $\square$

When $y =$ _____, $0.5y =$ _____.

D $6k; k = \frac{1}{3}$ (Hint: Think of 6 as $\frac{6}{1}$.)

Substitute $\frac{1}{3}$ for *k*. $6\left(\frac{\square}{\square}\right)$

Multiply. $\square$

When $k =$ _____, $6k =$ _____.

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EXPLORE ACTIVITY *(cont'd)*

YOUR TURN

Evaluate each expression for the given value of the variable.

1. $4x; x = 8$ _____ **2.** $6.5 - n; n = 1.8$ _____ **3.** $\frac{m}{6}; m = 18$ _____

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Online Assessment and Intervention.
Your Turn

Math On the Spot

Personal Math Trainer
Online Assessment and Intervention.
Interactive Example

Using the Order of Operations

Expressions may have more than one operation or more than one variable. To evaluate these expressions, substitute the given value for each variable and then use the order of operations.

EXAMPLE 2

COMMON CORE 6.EE.2c

Evaluate each expression for the given value of the variable.

A $4(x - 4); x = 7$

$4(7 - 4)$	Substitute 7 for x.
$4(3)$	Subtract inside the parentheses.
12	Multiply.

When $x = 7$, $4(x - 4) = 12$.

B $4x - 4; x = 7$

$4(7) - 4$	Substitute 7 for x.
$28 - 4$	Multiply.
24	Subtract.

When $x = 7$, $4x - 4 = 24$.

C $w - x + y; w = 6, x = 5, y = 3$

$(6) - (5) + (3)$	Substitute 6 for w, 5 for x, and 3 for y.
$1 + 3$	Subtract.
4	Add.

When $w = 6$, $x = 5$, $y = 3$, $w - x + y = 4$.

D $x^2 - x; x = 9$

$(9)^2 - (9)$	Substitute 9 for each x.
$81 - 9$	Evaluate exponents.
72	Subtract.

When $x = 9$, $x^2 - x = 72$.

Math Talk
Mathematical Practices
Is $w - x + y$ equivalent to $w - y + x$? Explain any difference in the order the math operations are performed.

YOUR TURN

Evaluate each expression for $n = 5$.

4. $3(n + 1)$ _____ **5.** $4(n - 4) + 14$ _____ **6.** $6n + n^2$ _____

Evaluate each expression for $a = 3$, $b = 4$, and $c = 6$.

7. $ab - c$ _____ **8.** $bc + 5a$ _____ **9.** $a^3 - (b + c)$ _____

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Online Assessment and Intervention.
Your Turn

Math On the Spot

Personal Math Trainer
Online Assessment and Intervention.
Interactive Example

Evaluating Real-World Expressions

You can evaluate expressions to solve real-world problems.

EXAMPLE 3

Real World

COMMON CORE 6.EE.2c

The expression $1.8c + 32$ gives the temperature in degrees Fahrenheit for a given temperature in degrees Celsius c. Find the temperature in degrees Fahrenheit that is equivalent to 30 °C.

STEP 1 Find the value of c.

$c = 30$ °C

STEP 2 Substitute the value into the expression.

$1.8c + 32$	
$1.8(30) + 32$	Substitute 30 for c.
$54 + 32$	Multiply.
86	Add.

86 °F is equivalent to 30 °C.

YOUR TURN

10. The expression $6x^2$ gives the surface area of a cube, and the expression x^3 gives the volume of a cube, where x is the length of one side of the cube. Find the surface area and the volume of a cube with a side length of 2 m.

$S =$ _____ m^2; $V =$ _____ m^3

11. The expression $60m$ gives the number of seconds in m minutes. How many seconds are there in 7 minutes?

_____ seconds

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Online Assessment and Intervention.
Your Turn

Guided Practice

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Selected Answers — See all the selected answers.

Evaluate each expression for the given value(s) of the variable(s). (Explore Activity Example 1 and Example 2)

1. $x - 7; x = 23$ _____
2. $3a - b; a = 4, b = 6$ _____
3. $\frac{8}{t}; t = 4$ _____
4. $9 + m; m = 1.5$ _____
5. $\frac{1}{2}w + 2; w = \frac{1}{9}$ _____
6. $5(6.2 + z); z = 3.8$ _____

7. The table shows the prices for games in Bella's soccer league. Her parents and grandmother attended a soccer game. How much did they spend if they all went together in one car? (Example 3)

Women's Soccer Game Prices	
Student tickets	$6
Nonstudent tickets	$12
Parking	$5

   a. Write an expression that represents the cost of one carful of nonstudent soccer fans. Use x as the number of people who rode in the car and attended the game.

   _____ is an expression that represents the cost of one carful of nonstudent soccer fans.

   b. Since there are three attendees, evaluate the expression $12x + 5$ for $x = 3$.

   12(____) + 5 = _____ + 5 = _____

   The family spent _____ to attend the game.

8. Stan wants to add trim all around the edge of a rectangular tablecloth that measures 5 feet long by 7 feet wide. The perimeter of the rectangular tablecloth is twice the length added to twice the width. How much trim does Stan need to buy? (Example 3)

   a. Write an expression that represents the perimeter of the rectangular tablecloth. Let l represent the length of the tablecloth and w represent its width. The expression would be _____.

   b. Evaluate the expression $P = 2w + 2l$ for $l = 5$ and $w = 7$.

   2(_____) + 2(_____) = 14 + _____ = _____

   Stan bought _____ of trim to sew onto the tablecloth.

ESSENTIAL QUESTION CHECK-IN

9. How do you know the correct order in which to evaluate algebraic expressions?

_____

_____

Name _____ Class _____ Date _____

10.2 Independent Practice

COMMON CORE 6.EE.2c

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Selected Answers — See all the selected answers.

10. The table shows ticket prices at the Movie 16 theater. Let a represent the number of adult tickets, c the number of children's tickets, and s the number of senior citizen tickets.

Movie 16 Ticket Prices	
Adults	$8.75
Children	$6.50
Seniors	$6.50

   a. Write an expression for the total cost of tickets.

   _____

   _____

   b. The Andrews family bought 2 adult tickets, 3 children's tickets, and 1 senior ticket. Evaluate your expression in part a to find the total cost of the tickets.

   _____

   _____

   c. The Spencer family bought 4 adult tickets and 2 children's tickets. Did they spend the same as the Andrews family? Explain.

   _____

11. The area of a triangular sail is given by the expression $\frac{1}{2}bh$, where b is the length of the base and h is the height. What is the area of a triangular sail in a model sailboat when $b = 12$ inches and $h = 7$ inches?

    $A =$ _____ in.^{2}

12. Ramon wants to balance his checking account. He has $2,340 in the account. He writes a check for $140. He deposits a check for $268. How much does Ramon have left in his checking account? _____

13. **Look for a Pattern** Evaluate the expression $6x - x^2$ for $x = 0, 1, 2, 3, 4, 5,$ and 6. Use your results to fill in the table and describe any pattern that you see.

x	0	1	2	3	4	5	6
$6x - x^2$							

_____

_____

_____

_____

_____

_____

_____

14. The kinetic energy (in joules) of a moving object can be calculated from the expression $\frac{1}{2}mv^2$, where m is the mass of the object in kilograms and v is its speed in meters per second. Find the kinetic energy of a 0.145-kg baseball that is thrown at a speed of 40 meters per second.

    $E =$ _____ joules

15. The area of a square is given by x^2, where x is the length of one side. Mary's original garden was in the shape of a square. She has decided to double the area of her garden. Write an expression that represents the area of Mary's new garden. Evaluate the expression if the side length of Mary's original garden was 8 feet.

_____

_____

LESSON
10.3 Generating Equivalent Expressions

COMMON CORE 6.EE.3
Apply the properties of operations to generate equivalent expressions. *Also 6.EE.2b, 6.EE.4*

ESSENTIAL QUESTION

How can you identify and write equivalent expressions?

EXPLORE ACTIVITY 1

COMMON CORE 6.EE.4

Identifying Equivalent Expressions

One way to test whether two expressions might be equivalent is to evaluate them for the same value of the variable.

Match the expressions in List A with their equivalent expressions in List B.

List A	List B
$5x + 65$	$5x + 1$
$5(x + 1)$	$5x + 5$
$1 + 5x$	$5(13 + x)$

A Evaluate each of the expressions in the lists for $x = 3$.

List A	List B
$5(3) + 65 =$ ☐	$5(3) + 1 =$ ☐
$5(3 + 1) =$ ☐	$5(3) + 5 =$ ☐
$1 + 5(3) =$ ☐	$5(13 + 3) =$ ☐

B Which pair(s) of expressions have the same value for $x = 3$?

_____

_____

C How could you further test whether the expressions in each pair are equivalent?

_____

D Do you think the expressions in each pair are equivalent? Why or why not?

_____

_____

EXPLORE ACTIVITY 1 *(cont'd)*

Reflect

1. **Error Analysis** Lisa evaluated the expressions $2x$ and x^2 for $x = 2$ and found that both expressions were equal to 4. Lisa concluded that $2x$ and x^2 are equivalent expressions. How could you show Lisa that she is incorrect?

_____

_____

EXPLORE ACTIVITY 2

COMMON CORE 6.EE.3

Modeling Equivalent Expressions

You can also use models to determine if two expressions are equivalent. *Algebra tiles* are one way to model expressions.

Algebra Tiles
+ = 1
+ = x

Determine if the expression $3(x + 2)$ is equivalent to $3x + 6$.

A Model each expression using algebra tiles.

$3(x + 2)$ $\qquad$ $3x + 6$

B The model for $3(x + 2)$ has _____ x tiles and _____ 1 tiles.

The model for $3x + 6$ has _____ x tiles and _____ 1 tiles.

C Is the expression $3(x + 2)$ equivalent to $3x + 6$? Explain.

_____

_____

Reflect

2. Use algebra tiles to determine if $2(x + 3)$ is equivalent to $2x + 3$. Explain your answer.

_____

_____

Writing Equivalent Expressions Using Properties

Properties of operations can be used to identify equivalent expressions.

Properties of Operations	Examples
Commutative Property of Addition: When adding, changing the order of the numbers does not change the sum.	$3 + 4 = 4 + 3$
Commutative Property of Multiplication: When multiplying, changing the order of the numbers does not change the product.	$2 \times 4 = 4 \times 2$
Associative Property of Addition: When adding more than two numbers, the grouping of the numbers does not change the sum.	$(3 + 4) + 5 = 3 + (4 + 5)$
Associative Property of Multiplication: When multiplying more than two numbers, the grouping of the numbers does not change the product.	$(2 \times 4) \times 3 = 2 \times (4 \times 3)$
Distributive Property: Multiplying a number by a sum or difference is the same as multiplying by each number in the sum or difference and then adding or subtracting.	$6(2 + 4) = 6(2) + 6(4)$ $8(5 - 3) = 8(5) - 8(3)$
Identity Property of Addition: Adding zero to a number does not change its value.	$9 + 0 = 9$
Identity Property of Multiplication: Multiplying a number by one does not change its value.	$1 \times 7 = 7$

EXAMPLE 1

COMMON CORE 6.EE.3

Use a property to write an expression that is equivalent to $x + 3$.

The operation in the expression is addition.

You can use the Commutative Property of Addition to write an equivalent expression: $x + 3 = 3 + x$.

YOUR TURN

For each expression, use a property to write an equivalent expression. Tell which property you used.

3. $(ab)c =$ _____

4. $3y + 4y =$ _____

5. 6×7 _____

Identifying Equivalent Expressions Using Properties

EXAMPLE 2

COMMON CORE 6.EE.3

Use the properties of operations to determine if the expressions are equivalent.

A $3(x - 2); 3x - 6$

$3(x - 2) = 3x - 6$ Distributive Property

$3(x - 2)$ and $3x - 6$ are equivalent expressions.

B $2 + x; \frac{1}{2}(4 + x)$

$\frac{1}{2}(x + 4) - \frac{1}{2}x + 2$ Distributive Property

$= 2 + \frac{1}{2}x$ Commutative Property

$2 + x$ does not equal $2 + \frac{1}{2}x$.

They are not equivalent expressions.

YOUR TURN

Use the properties of operations to determine if the expressions are equivalent.

6. $6x - 8; 2(3x - 5)$ _____

7. $2 - 2 + 5x; 5x$ _____

8. Jamal bought 2 packs of stickers and 8 individual stickers. Use x to represent the number of stickers in a pack of stickers and write an expression to represent the number of stickers Jamal bought. Is the expression equivalent to $2(4 + x)$? Check your answer with algebra tile models.

Generating Equivalent Expressions

Parts of an algebraic expression		
terms	The parts of the expression that are separated by + or − signs	$12 + 3y^2 + 4x + 2y^2 + 4$
coefficients	Numbers that are multiplied by at least one variable	$12 + 3y^2 + 4x + 2y^2 + 4$
like terms	Terms with the same variable(s) raised to the same power(s)	$12 + 3y^2 + 4x + 2y^2 + 4$

When an expression contains like terms, you can use properties to combine the like terms and write an equivalent expression.

EXAMPLE 3

COMMON CORE 6.EE.3, 6.EE.2b

Combine like terms.

A $6x^2 - 4x^2$

$6x^2$ and $4x^2$ are like terms.

$6x^2 - 4x^2 = x^2(6 - 4)$ — Distributive Property

$= x^2(2)$ — Subtract inside the parentheses.

$= 2x^2$ — Commutative Property of Multiplication

$6x^2 - 4x^2 = 2x^2$

B $3a + 2(b + 5a)$

$3a + 2(b + 5a) = 3a + 2b + 2(5a)$ — Distributive Property

$= 3a + 2b + (2 \cdot 5)a$ — Associative Property of Multiplication

$= 3a + 2b + 10a$ — Multiply 2 and 5.

$= 3a + 10a + 2b$ — Commutative Property of Addition

$= (3 + 10)a + 2b$ — Distributive Property

$= 13a + 2b$ — Add inside the parentheses.

$3a + 2(b + 5a) = 13a + 2b$

C $y + 11x + 7y - 7x$

y and $7y$ are like terms; $11x$ and $7x$ are like terms.

$y + 11x + 7y - 7x = y + 7y + 11x - 7x$ — Commutative Property

$= (1 + 7)y + (11 - 7)x$ — Distributive Property

$= 8y + 4x$ — Simplify inside parentheses.

$y + 11x + 7y - 7x = 8y + 4x$

Math Talk

Mathematical Practices

Write 2 terms that can be combined with $7y^4$.

Math On the Spot

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Interactive Example

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Online Assessment and Intervention

Your Turn

Personal Math Trainer

Online Assessment and Intervention

Independent Practice

Selected Answers

See all the selected answers.

YOUR TURN

Combine like terms.

9. $8y - 3y =$ _____

10. $6x^2 + 4(x^2 - 1) =$ _____

11. $4a^5 - 2a^5 + 4b + b =$ _____

12. $8m + 14 - 12 + 4n =$ _____

Guided Practice

1. Evaluate each of the expressions in the list for $y = 5$. Then, draw lines to match the expressions in List A with their equivalent expressions in List B. (Explore Activity 1)

List A	List B
$4 + 4y =$ _____	$4y - 4 =$ _____
$4(y - 1) =$ _____	$4(y + 1) =$ _____
$4y + 1 =$ _____	$1 + 4y =$ _____

2. Determine if the expressions are equivalent by comparing the models. (Explore Activity 2) _____

For each expression, use a property to write an equivalent expression. Tell which property you used. (Example 1)

3. $ab =$ _____

4. $5(3x - 2) =$ _____

Use the properties of operations to determine if each pair of expressions is equivalent. (Example 2)

5. $\frac{1}{2}(4 - 2x); 2 - 2x$ _____

6. $\frac{1}{2}(6x - 2); 3 - x$ _____

Combine like terms. (Example 3)

7. $32y + 12y =$ _____

8. $12 + 3x - x - 12 =$ _____

ESSENTIAL QUESTION CHECK-IN

9. Describe two ways to write equivalent algebraic expressions.

_____

_____

Name _____ Class _____ Date _____

10.3 Independent Practice

COMMON CORE 6.EE.2b, 6.EE.3, 6.EE.4

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Selected Answers

For each expression, use a property to write an equivalent expression. Tell which property you used.

10. $cd =$ _____

11. $x + 13 =$ _____

12. $4(2x - 3) =$ _____

13. $2 + (a + b) =$ _____

14. Draw algebra tile models to prove that $4 + 8x$ and $4(2x + 1)$ are equivalent.

Combine like terms.

15. $7x^4 - 5x^4 =$ _____

16. $32y + 5y =$ _____

17. $6b + 7b - 10 =$ _____

18. $2x + 3x + 4 =$ _____

19. $y + 4 + 3(y + 2) =$ _____

20. $7a^2 - a^2 + 16 =$ _____

21. $3y^2 + 3(4y^2 - 2) =$ _____

22. $z^2 + z + 4z^3 + 4z^2 =$ _____

23. $0.5(x^4 - 3) + 12 =$ _____

24. $\frac{1}{4}(16 + 4p) =$ _____

25. **Justify Reasoning** Determine whether $3x + 12 + x$ is equivalent to $4(3 + x)$. Use properties of operations to justify your answer.

26. William earns $13 an hour working at a movie theater. Last week he worked h hours at the concession stand and three times as many hours at the ticket counter. Write and simplify an expression for the amount of money William earned last week.

Going Further 10.3

Equivalent Expressions

COMMON CORE 6.EE.1

Write and evaluate numerical expressions involving whole-number exponents. *Also 6.EE.2, 6.EE.2c, 6.EE.3, 6.EE.4*

ESSENTIAL QUESTION

How can you determine if two expressions are equivalent?

EXPLORE ACTIVITY COMMON CORE 6.EE.1, 6.EE.2, 6.EE.2c

Substituting to Determine Equivalence

Expressions can be equivalent for all values or only when specific values are used. To determine if expressions are equivalent when given a specific value, substitute the value into the expressions and evaluate the expressions.

Substitute the given value into the expressions to determine if they are equivalent for the given value. Explain your reasoning.

A $13h + 39h$ and $13h + 13(3h)$ when $h = 4$

	$13h + 39h$		$13h + 13(3h)$
Substitute.	$13(\_\_\_) + 39(\_\_\_)$	Substitute.	$13(\_\_\_) + 13(3 \times \_\_\_)$
Multiply.	_____ + _____	Multiply.	_____ + $13(\_\_\_)$
Add.	_____	Multiply.	_____ + _____
		Add.	_____

The two expressions [are / are not] equivalent when $h = 4$. When 4 is substituted in $13h + 39h$, the result is _____. When 4 is substituted in $13h + 13(3h)$, the result is _____.

B $3(p + 6) - 4$ and $3p + 2^2$ when $p = 5$

	$3(p + 6) - 4$		$3p + 2^2$
Substitute 5 for p.	$3(\_\_\_ + 6) - 4$	Substitute 5 for p.	$3(\_\_\_) + 2^2$
Add.	$3(\_\_\_) - 4$	Evaluate the exponent.	$3(\_\_\_) + (\_\_\_)$
Multiply.	_____ − 4	Multiply.	_____ + _____
Subtract.	_____	Add.	_____

The two expressions [are/ are not] equivalent for $p = 5$. When 5 is substituted in $3(p + 6) - 4$, the result is _____. When 5 is substituted in $3p + 2^2$, the result is _____.

Simplifying To Determine Equivalence

You can determine if two expressions are equivalent by simplifying the expressions.

EXAMPLE

COMMON CORE 6.EE.3, 6.EE.4

Determine if the expressions are equivalent. Justify your reasoning.

A $4a + 4(6a + 7b) + 7b$ and $(4a + 5b)7$

$4a + 4(6a + 7b) + 7b$		$(4a + 5b)7$	
$4a + 4(6a) + 4(7b) + 7b$	Distributive Property	$7(4a + 5b)$	Commutative Property
$4a + 24a + 28b + 7b$	Multiply.	$7(4a) + 7(5b)$	Distributive Property
$28a + 35b$	Combine like terms.	$28a + 35b$	Multiply.

The expressions are equivalent.

B $3b^3 + 8 - 2c - b^3 + 6c$ and $2(2b^3 + 4c) + 8$

$3b^3 + 8 - 2c - b^3 + 6c$		$2(2b^3 + 4c) + 8$	
$3b^3 - b^3 - 2c + 6c + 8$	Commutative Property	$4b^3 + 8c + 8$	Distributive Property
$2b^3 + 4c + 8$	Combine like terms.		

The expressions are not equivalent.

Practice

Determine if the expressions are equivalent for the given value. Explain your reasoning.

1. $\frac{p + p}{2}$ and $2p - 6$ when $p = 6$

2. $7(2m + 4)$ and $10m + 4m + 14$ when $m = 3$

Determine if the expressions are equivalent by simplifying. Justify your answer.

3. $2(4x^2 + 4) + 10x^2 + 1$ and $6(x^2 + 1) + 12x^2 + 3$

4. $5(x + 2) + 4 + 10x$ and $8x - 9 + 8x + 23$

MODULE QUIZ

Ready to Go On?

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Ready to Go On?

10.1 Modeling and Writing Expressions

Write each phrase as an algebraic expression.

1. p divided by 6 _____
2. 65 less than j _____
3. the sum of 185 and h _____
4. the product of 16 and g _____
5. Let x represent the number of television show episodes that are taped in a season. Write an expression for the number of episodes taped in 4 seasons. _____

10.2 Evaluating Expressions

Evaluate each expression for the given value of the variable.

6. $8p; p = 9$ _____
7. $11 + r; r = 7$ _____
8. $4(d + 7); d = -2$ _____
9. $\frac{60}{m}; m = 5$ _____
10. To find the area of a triangle, you can use the expression $b \times h \div 2$, where b is the base of the triangle and h is its height. What is the area of a triangle with a base of 6 and a height of 8? _____

10.3 Generating Equivalent Expressions

11. Draw lines to match the expressions in List A with their equivalent expressions in List B.

List A	List B
$7x + 14$	$7(1 + x)$
$7 + 7x$	$7x - 7$
$7(x - 1)$	$7(x + 2)$

ESSENTIAL QUESTION

12. How can you solve problems involving equivalent expressions?

COMMON CORE

MODULE 10 MIXED REVIEW

Assessment Readiness

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Selected Response

1. Which expression represents the product of 83 and x?

   Ⓐ $83 + x$
   Ⓑ $83 \div x$
   Ⓒ $83x$
   Ⓓ $83 - x$

2. Which phrase describes the algebraic expression $\frac{r}{9}$?

   Ⓐ the product of r and 9
   Ⓑ the quotient of r and 9
   Ⓒ 9 less than r
   Ⓓ r more than 9

3. Rhonda was organizing photos in a photo album. She took 60 photos and divided them evenly among p pages. Which algebraic expression represents the number of photos on each page?

   Ⓐ $p - 60$
   Ⓑ $60 - p$
   Ⓒ $\frac{p}{60}$
   Ⓓ $\frac{60}{p}$

4. Using the algebraic expression $4n + 6$, what is the greatest whole-number value of n that will give you a result less than 100?

   Ⓐ 22
   Ⓑ 23
   Ⓒ 24
   Ⓓ 25

5. Evaluate $7w - 14$ for $w = 9$.

   Ⓐ 2
   Ⓑ 18
   Ⓒ 49
   Ⓓ 77

6. Katie has read 32% of a book. If she has read 80 pages, how many more pages does Katie have left to read?

   Ⓐ 40
   Ⓑ 170
   Ⓒ 200
   Ⓓ 250

7. The expression $12(x + 4)$ represents the total number of CDs Mei bought in April and May at $12 each. Which property is applied to write the equivalent expression $12x + 48$?

   Ⓐ Associative Property of Addition
   Ⓑ Associative Property of Multiplication
   Ⓒ Commutative Property of Multiplication
   Ⓓ Distributive Property

Mini-Task

8. You can convert a temperature given in degrees Celsius to a Fahrenheit temperature by using the expression $9x \div 5 + 32$, where x is the Celsius temperature.

   a. Water freezes when the temperature is 0 °C. At what Fahrenheit temperature does water freeze? _____

   b. Water boils at 100 °C. At what temperature does water boil in degrees Fahrenheit? _____

   c. The temperature of some water is 15 °C. What is the Fahrenheit temperature? _____

UNIT 4

Study Guide Review

MODULE 9

Generating Equivalent Numerical Expressions

Key Vocabulary
base *(base (en numeración))*
exponent *(exponente)*
order of operations *(orden de las operaciones)*
power *(potencia)*

ESSENTIAL QUESTION

How can you generate equivalent numerical expressions and use them to solve real-world problems?

EXAMPLE 1

Find the value of each power.

A. 0.9^2

$0.9^2 = 0.9 \times 0.9 = 0.81$

B. 18^0

Any number raised to the power of 0 is 1.

$18^0 = 1$

C. $\left(\frac{1}{4}\right)^4$

$\left(\frac{1}{4}\right)^4 = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{256}$

EXAMPLE 2

Find the prime factorization of 60.

2	60
2	30
3	15
5	5
	1

$60 = 2 \times 2 \times 3 \times 5$

$60 = 2^2 \times 3 \times 5$

The prime factorization of 60 is $2^2 \times 3 \times 5$.

EXAMPLE 3

Simplify each expression.

A. $4 \times (2^3 + 5)$

$= 4 \times (8 + 5)$ $2^3 = 8$

$= 4 \times 13$ Add.

$= 52$ Multiply.

B. $27 \div 3^2 \times 6$

$= 27 \div 9 \times 6$ $3^2 = 9$

$= 3 \times 6$ Divide.

$= 18$ Multiply.

EXERCISES

Use an exponent to write each expression. (Lesson 9.1)

1. 3.6×3.6 _____
2. $9 \times 9 \times 9 \times 9$ _____
3. $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$ _____

Find the value of each power. (Lesson 9.1)

4. 12^0 _____ **5.** 13^2 _____ **6.** $\left(\frac{2}{7}\right)^3$ _____

7. 0.4^2 _____ **8.** $\left(\frac{4}{9}\right)^1$ _____ **9.** 0.7^3 _____

Find the prime factorization of each number. (Lesson 9.2)

10. 75 _____ **11.** 29 _____ **12.** 168 _____

13. Eduardo is building a sandbox that has an area of 84 square feet. What are the possible whole number measurements for the length and width of the sandbox? (Lesson 9.2)

_____

Simplify each expression. (Lesson 9.3)

14. $2 \times 5^2 - (4 + 1)$ _____ **15.** $\frac{22 - (3^2 + 4)}{12 \div 4}$ _____

MODULE 10

Generating Equivalent Algebraic Expressions

ESSENTIAL QUESTION

How can you generate equivalent algebraic expressions and use them to solve real-world problems?

Key Vocabulary

- algebraic expression *(expresión algebraica)*
- coefficients *(coeficiente)*
- constant *(contante)*
- equivalent expressions *(expresión equivalente)*
- evaluating *(evaluar)*
- term *(término (en una expresión))*

EXAMPLE 1

Evaluate each expression for the given values of the variables.

A. $2(x^2 - 9); x = 5$

$2(5^2 - 9)$ — $5^2 = 25$

$= 2(16)$ — Subtract.

$= 32$ — Multiply.

When $x = 5$, $2(x^2 - 9) = 32$.

B. $w + y^2 + 3w; w = 2, y = 6$

$2 + 6^2 + 3(2)$ — $6^2 = 36$

$= 2 + 36 + 6$ — Multiply.

$= 44$ — Add.

When $w = 2$ and $y = 6$, $w + y^2 + 3w = 44$.

EXAMPLE 2

Determine whether the algebraic expressions are equivalent: $5(x + 2)$ and $10 + 5x$.

$5(x + 2) = 5x + 10$ — Distributive Property

$= 10 + 5x$ — Commutative Property

$5(x + 2)$ is equal to $10 + 5x$. They are equivalent expressions.

EXERCISES

Write each phrase as an algebraic expression. (Lesson 10.1)

1. x subtracted from 15 _____ **2.** 12 divided by t _____

3. 4 groups of y _____ **4.** the sum of z and 7 _____

Write a phrase for each algebraic expression. (Lesson 10.1)

5. $8p$ _____

6. $s + 7$ _____

Evaluate each expression for the given values of the variables. (Lesson 10.2)

7. $8z + 3; z = 8$ _____ **8.** $3(7 + x^2); x = 2$ _____

9. $s - 5t + s^2; s = 4, t = -1$ _____ **10.** $x - y^3; x = -7, y = 3$ _____

11. The expression $\frac{1}{2}(h)(b_1 + b_2)$ gives the area of a trapezoid, with b_1 and b_2 representing the two base lengths of a trapezoid and h representing the height. Find the area of a trapezoid with base lengths 4 in. and 6 in. and a height of 8 in. (Lesson 10.2)

_____

Determine if the expressions are equivalent. (Lesson 10.3)

12. $7 + 7x; 7\left(x + \frac{1}{7}\right)$ _____

13. $2.5(3 + x); 2.5x + 7.5$ _____

Combine like terms. (Lesson 10.3)

14. $3m - 6 + m^2 - 5m + 1$ _____

15. $7x + 4(2x - 6)$ _____

16. $b^2 + 3 + 2b^2 + 4 - 7$ _____

17. $3(p + 5) - 8 + 11p$ _____

COMMON CORE

UNIT 4 MIXED REVIEW

Assessment Readiness

Selected Response

1. Which expression is equivalent to $2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3$?

Ⓐ 2.3×5

Ⓑ 23^5

Ⓒ $2^5 \times 3^5$

Ⓓ 2.3^5

2. Which operation should you perform first when you simplify $63 - (2 + 54 \times 6) \div 5$?

Ⓐ addition

Ⓑ division

Ⓒ multiplication

Ⓓ subtraction

3. Sheena was organizing items in a scrapbook. She took 25 photos and divided them evenly between p pages. Which algebraic expression represents the number of photos on each page?

Ⓐ $p - 25$

Ⓑ $25 - p$

Ⓒ $\frac{p}{25}$

Ⓓ $\frac{25}{p}$

4. Which is another way to write $7 \times 7 \times 7 \times 7$?

Ⓐ 7^4

Ⓑ $7(4)$

Ⓒ 28

Ⓓ 4^7

5. Angela earns x dollars an hour. On Friday, she worked 6 hours. On Saturday, she worked 8 hours. Which expression shows how much she earned both days?

Ⓐ $6x + 8$

Ⓑ $8x \times 6x$

Ⓒ $(6 + 8)x$

Ⓓ $\frac{6 + 8}{x}$

6. Marcus is doing a science experiment in which he measures the rate at which bacteria multiply. Every 15 seconds, the bacteria double in number. If there are 10 bacteria now, how many will there be in 2 minutes?

Ⓐ 160 bacteria

Ⓑ 256 bacteria

Ⓒ 1,280 bacteria

Ⓓ 2,560 bacteria

7. The prime factorization of which number is $2^5 \times 5$?

Ⓐ 50

Ⓑ 125

Ⓒ 160

Ⓓ 500

8. Which expression has a value of 36 when $x = 4$ and $y = 7$?

Ⓐ $2xy$

Ⓑ $2x + 4y$

Ⓒ $6y - x$

Ⓓ $12x - 2y$

Hot Tip! **When possible, use logic to eliminate at least two answer choices.**

9. What should you do first to simplify the expression $(4^3 + 9) \div 76 + 5$?

Ⓐ Add 4 and 9.

Ⓑ Add 76 and 5.

Ⓒ Multiply $4 \times 4 \times 4$.

Ⓓ Divide $(4^3 + 9)$ by 76.

10. Which ratio is equivalent to 4:10?

Ⓐ $\frac{2}{5}$

Ⓑ $\frac{8}{10}$

Ⓒ $\frac{12}{16}$

Ⓓ $\frac{16}{10}$

11. Travis and Paula went to lunch. Travis ordered a sandwich for $7.50, and Paula ordered a burger for $5.25. After lunch, they left a 15% tip for their waiter. How much money did they spend altogether?

Ⓐ $12.75

Ⓑ $14.66

Ⓒ $15.95

Ⓓ $16.00

12. Which shows the following numbers in order from greatest to least?

$1.5, \frac{2}{4}, \frac{4}{2}, 1.05$

Ⓐ $\frac{4}{2}, 1.5, 1.05, \frac{2}{4}$

Ⓑ $1.05, 1.5, \frac{2}{4}, \frac{4}{2}$

Ⓒ $\frac{4}{2}, \frac{2}{4}, 1.5, 1.05$

Ⓓ $1.05, \frac{4}{2}, \frac{2}{4}, 1.5$

Mini-Tasks

13. For every bag of trail mix the local Scout Guide troop sells, they earn $0.45.

a. Write an expression to represent this situation.

b. Sarah sold 52 bags of trail mix. How much did she earn for her troop?

c. Let x represent the total number of bags of trail mix sold by Sarah's troop. Write an expression to show what percentage of bags Sarah sold.

14. Robert is replacing sod in two square-shaped areas of his backyard. One side of the first area is 7.5 feet. One side of the other area is 5.7 feet. The sod costs y dollars per square foot.

a. Write an expression to show how much Robert will spend on sod.

b. If the sod costs $3.25 per square foot, about how much will Robert spend to put sod down in both areas of his backyard? Round to the nearest dollar.

15. Jose wants to find how many gallons of water he needs to fill his cube-shaped aquarium. One side of his aquarium is 4 feet long.

a. Write and solve an expression to find the volume of Jose's aquarium.

b. One cubic foot is equal to 7.48 gallons of water. How many gallons of water does Jose need to fill his aquarium? Round to the nearest gallon.